A Novel Neural Architecture with High Storage Capacity

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Index terms: associative memory, fuzzy neural networks

Abstract: A novel method of pattern recognition using Polynomial Bidirectional Hetero-Correlator (PBHC) is proposed. Simulation results show that the new scheme displays superior storage capacity over other BAM-like associative memories and fuzzy associative memories.

Introduction: The BAM-like (Bidirectional associative memory-like) associative memory is a two-layer hetero-associator that stores a set of bipolar pairs. Owing to their easiness to be encoded and high noise immunity, BAM's are well-suited for pattern recognition, intelligent control and optimization problems. The original Kosko's BAM suffers from low storage capacity [2]. Thus, many efforts have been made to improve the performance of Kosko's BAM [1], [3], [5]. Some of these models strengthen the BAM architecture by using the Hamming stability learning algorithm (SBAM) [5], the asymmetrical BAM model (ABAM) [3], or introducing a general model of BAM (GBAM) to improve the performance [1]. Kosko's fuzzy associative memory (FAM) is the very first example to use neural networks to articulate fuzzy rules for fuzzy systems. Despite its simplicity and modularity, his model suffers from extremely low memory capacity, i.e., one rule per FAM matrix. Besides, it is limited to small rule-based applications. Chung and Lee [4] proposed a multiple-rule storage method of a FAM matrix. They showed that more than one rule can be encoded by Kosko's FAM. However, the actual capacity will depend on the dimension of the matrix and the rule characteristics, e.g., how many the rules are overlapped. The capacity of this model depends on whether the membership function is semi-overlapped or not. In this letter, we present a novel method of fuzzy data processing using polynomial bidirectional hetero-correlator (PBHC). A perfect recall theorem is established and the implementation of the PBHC model is more efficient accordingly. This has a higher capacity for pattern pair storage than that of the conventional BAMs and fuzzy memories.

0-7695-0619-4/00 $10.00 © 2000 IEEE

Proposed polynomial bidirectional hetero-associator: Assume that we are given M pattern pairs, which are $\{(X_1,Y_1),(X_2,Y_2),\ldots,(X_M,Y_M)\}$, where $X_i = (x_{i1},x_{i2},\ldots,x_{in})$, $Y_i = (y_{i1},y_{i2},\ldots,y_{ip})$. Let $1 \le i \le M$, $x_{ij} \in [0,1]$, $1 \le j \le n$, $y_{ij} \in [0,1]$, $1 \le j \le p$, n and p are the component dimensions of X_i and Y_i, and n is assumed to be smaller than or equal to p without any loss of generality. x_{ij}, $y_{ij} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$, fuzzy space = [1,0], λ is a fuzzy quantum, and σ is a fuzzy quantum gap. For instance, by assuming that $\lambda = 10$, $\sigma = 1/\lambda = 0.1$ can be obtained. We use the following evolution equations in the recall process of the PBHC,

$$y_k = H\left(\sum_{i=1}^{M} y_{ik} \cdot \left((u - \|X_i - X\|^2)/u\right)^{M^Z} \Big/ \sum_{i=1}^{M} \left((u - \|X_i - X\|^2)/u\right)^{M^Z}\right) \tag{1}$$

$$x_k = H\left(\sum_{i=1}^{M} x_{ik} \cdot \left((u - \|Y_i - Y\|^2)/u\right)^{M^Z} \Big/ \sum_{i=1}^{M} \left((u - \|Y_i - Y\|^2)/u\right)^{M^Z}\right) \tag{2}$$

where X_i, Y_i, $i = 1,\ldots,M$, represent the stored patterns, X or Y is the initial vector presented to the network, x_k and x_{ik} denote the kth digits of X and X_i, respectively, y_k and y_{ik} represent the kth digits of Y and Y_i, respectively, Z is a positive integer, u denotes a function defined as the following equation,

$$u = \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\|X_i - X_j\|^2 + \|Y_i - Y_j\|^2\right), \tag{3}$$

and $H(\cdot)$ is a staircase function shown as the following equation,

$$H(x) = \begin{cases} 0, & x < 1/(2\lambda) \\ \lfloor x + 1/(2\lambda) \rfloor & \text{elsewhere} \end{cases} \tag{4}$$

Note that if $\lambda \to \infty$, then $H(x) \approx x$, for $x \in [0,1]$. Besides, u is bounded according to Eqn.(3).

The SNR (Signal-to-Noise Ratio) approach is adopted herein to compute the capacity of PBHC. Let X_i and Y_i be the stored pattern pairs. Assume that X_1 is the input pattern pair and Y_1 is recalled expectantly. Substituting X_1 for X allows us to rewrite Eqn.(1) as (Note that if $\lambda \to \infty$, then $H(x) \approx x$, for $x \in [0,1]$.)

$$y_k = H\left(\sum_{i=1}^{M} y_{ik} \cdot \left((u - \|X_i - X\|^2)/u\right)^{M^Z} \Big/ \sum_{i=1}^{M} \left((u - \|X_i - X\|^2)/u\right)^{M^Z}\right)$$

$$\approx \sum_{i=1}^{M} y_{ik} \cdot \left((u - \|X_i - X\|^2)/u\right)^{M^Z} \Big/ \sum_{i=1}^{M} \left((u - \|X_i - X\|^2)/u\right)^{M^Z}$$

$$y_k \cdot \sum_{i=1}^{M} \left((u - \|X_i - X\|^2)/u\right)^{M^Z} = \sum_{i=1}^{M} y_{ik} \cdot \left((u - \|X_i - X\|^2)/u\right)^{M^Z}$$

$$= y_{1k} \cdot 1^2 + y_{2k} \cdot \left((u - \|X_2 - X_1\|^2)/u\right)^{M^Z} + \ldots + y_{Mk} \cdot \left((u - \|X_M - X_1\|^2)/u\right)^{M^Z} \tag{5}$$

The largest noise that can appear is that any X_i, $i \ne 1$, is just one component different from X_1, and the difference of the corresponding component is only $1/(2\lambda)$. Meanwhile, the other components of X_i and X_1 remain the same. For instance, $X_1 = (x_{11},x_{12},\ldots,x_{1n})$, and $X_i = (x_{11},x_{12},\ldots,x_{1n} \pm 1/(2\lambda))$, $i \ne 1$, where x_{ik}, $y_{ik} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$. The first term in the above equation corresponds to the signal, and the other terms are the noise. Besides the first term, the remaining terms are actually the sum of M-1 times of the noise of a single noise term, e.g., y_{2k}. From Eqn.(5), the following inequalities can be obtained:

$$\begin{aligned} y_k &\le y_k \cdot \sum_{i=1}^{M} \left(\left(u - \|X_i - X\|^2\right)/u \right)^{M^Z} \\ &\le y_{1k} \cdot 1^2 + \sum_{j=2}^{M} y_{jk} \cdot \left(\left(u - 1/\lambda^2\right)/u \right)^{M^Z} \\ &\le y_{1k} \cdot 1^2 + (M-1) y_{2k} \cdot \left(\left(u - 1/\lambda^2\right)/u \right)^{M^Z} \\ &= S + N_{\max}. \end{aligned}$$

Thus, the first term in the above equation is the signal, and the second term is viewed as the noise in the worst case. Let $y_{2k} = j/\lambda$, $j \in \{0,1,2,3,\ldots,\lambda\}$, $y_{2k} \in \{0/\lambda, 1/\lambda, 2/\lambda, \ldots, \lambda/\lambda\}$. The sufficient condition for recalling y_{1k} correctly is that the noise must be bounded as

$$y_{1k} - \frac{1}{2\lambda} < y_{1k} \cdot 1^2 + (M-1) \cdot y_{2k} \cdot \left(\left(u - 1/\lambda^2\right)/u \right)^{M^Z} < y_{1k} + \frac{1}{2\lambda} \tag{6}$$

Herein, we take $y_{2k} = j/\lambda$ into the above equation, then it can be simplified as follows,

$$\left| (M-1) \cdot \left(\left(u - 1/\lambda^2\right)/u \right)^{M^Z} \right| < \frac{1}{2j} \le \frac{1}{2} \tag{7}$$

The worst case will occur when j=1, and we deem the above equation as the sufficient condition for the PBHC to accurately recall any desired pattern. Since the value of polynomial in the equation of absolute value is positive, the minimal Z in the worst case for the PBHC is derived in the following,

$$Z \ge (1/lnM) \cdot ln\left(ln(1/2) / ln\left(\left(u - (1/\lambda^2)\right)/u \right) \right),$$

$$\text{where } u \le C_2^M \cdot (n+p) \le C_2^M \cdot (2n) \le M \cdot (M-1) \cdot n. \tag{8}$$

The above Eqn.(8) is the lower bound solution of Z, and according to Eqn.(7), the minimal capacity can be derived as follows,

$$M \ge \left(ln(1/2) / ln\left(\left(u - (1/\lambda^2)\right)/u \right) \right)^{Z^{-1}},$$

where $u = M \cdot (M-1) \cdot n$.

Results and conclusion: The BAM-like associative memory consists of two layers of neurons. One layer has n neurons and the other has p neurons. n is assumed to be less than or equal to p without any loss of robustness. To evaluate a BAM-like associative memory, the most important thing, perhaps, is its storage capacity. We consider that some randomly generated desired attractors can be generally stored each as a strong stable state in a BAM-like associative memory by the corresponding evolution equation [1]. We find that the storage capacity of the GBAM is a little greater than n, that of the SBAM or ABAM is closer or equal to n, and the capacity of the Kosko's rule is $\le 0.15n$. By contrast, the fuzzy data recognition using the PBHC provides a significantly high capacity of storage for patterns. Table 1 and Fig. 1 presents the capacity (M) for Kosko's BAM, SBAM, ABAM, GBAM and our PBHC with λ=2, respectively. Notably, we use PBHC with λ=2 in order to derive a fair comparison with other BAM-like designs which usually process either binary vectors or bipolar vectors. According to other simulation results, the capacity of PBHC exceeds more than $2n$ when λ is greater than 2 and grows more than linearly as n increases. For instance, suppose λ=10 (Z=3), when n is equal to 30 and 60, we can obtain the capacity equal to 40006 and 84059, respectively. Therefore, the PBHC provides an extremely high storage capacity for pattern pairs. The practical capacity of the PBHC in the worst case is estimated, thereby allowing us to predetermine the size of the PBHC.

Acknowledgements: This research was partially supported by National Science Council under grant NSC 89-2219-E-110-001, 89-2215-E-110-014 and 89-2215-E-110-017.

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Table 1: Capacity comparison of Kosko BAM, SBAM, ABAM, GBAM, and PBHC

Name \ n	20	40	60	80	100
Kosko BAM	3	5	7	9	10
SBAM	19	40	60	80	100
ABAM	20	40	60	80	100
GBAM	20	46	75	106	136
PBHC (λ=2)	55	110	166	221	277

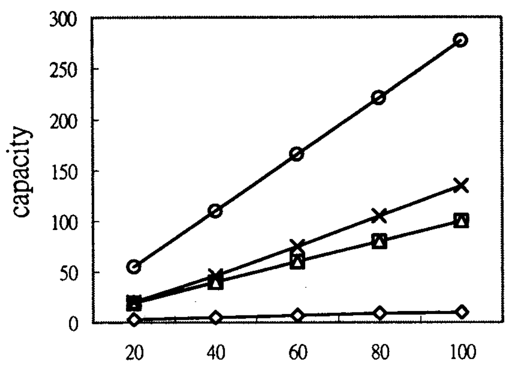

Fig. 1 *Capacity comparison of the BAM-like hetero-associators*