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BELIEF COMBINATION BY A POTENTIAL MODEL+

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ABSTRACT

A method of belief combination based on a heuristic potential model is proposed, in which a belief function associated with a piece of evidence is modeled as a belief density function and each point belief thereon generates a potential on the hypothesis. The procedure of belief combination is a spatial interpretation in which the influence of a piece of evidence on a hypothesis is based on the distance between the evidence and the hypothesis and the strength of the evidence. The resulted belief combination at the hypothesis is a cumulative integral of the potential generated by all point beliefs on the evidence. This model can handle both of discrete belief functions and continuous belief functions. Also it has resolved the conflicts resulting from either the mutual dependency relationship among different pieces of evidence or the structural dependency in an inference network due to various combination orders of evidence. A belief combination procedure for arbitrary number of evidence without any conflict is presented. Some examples are given to demonstrate the advantages of this method over the conventional approaches.

以一個位能模型作相信組合

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關鍵詞：推理、相信函數、相信組合、相互相依。

摘 要

本文提出一個作相信組合之位能模型，在其中一個證據之相信函數是以一個相信密度函數來表示，並且該證據上每一個點證據均在最後之結論上產生一個位能。進行相信組合之程序有一個空間上之解釋：一個證據對最後之結論的影響取決於它們之間的距離與該證據之強度，最後的相信組合是將各點證據所產生之位能積分起來。這個模型對分離性或連續性相信函數均可處理。它解決了一個由幾個相互相依的證據們所造成的或由一個推理網路中的結構性相依所造成的矛盾。本文也提出了一個可以組合任意數目的證據，並且無矛盾現象的程序。最後並以幾個例子來示範這個模型相較於傳統方法之優點。

1. INTRODUTION

Belief combination is one task of evidential reasoning which is referring to combination of relevant evidence for or against hypotheses, and is the core of many rule-based systems that will help people to do decision making, and diagnosis. The problem of belief combination due to many pieces of information or evidence conveying uncertainty, i.e., the evidence is sort of imprecise, incomplete, or vague, is worth focusing effect upon. The way to assess the hypothesis is to infer its it *belief value* from the belief values of the evidence. The belief

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value can be taken from a certain belief region, e.g., a unit interval [0,1], which can be discrete or continuous, or from a set of linguistic quantifiers, e.g., [*very unlikely, unlikely, likely, very likely*], etc. The fact that the *belief strength* of a piece of evidence is not subject to any change of another evidence is a basic definition of *independency* of the evidence. On the other hand, if the belief strength of one evidence is subject to other evidence, it is deemed that there is a *mutual dependency* relationship among these evidence, which can be totally dependent or partially dependent. If a hypothesis is supported by many pieces of evidence, then the combined belief strength of the hypothesis is the belief value caused not only by the individual evidence but also by the mutual dependencies between the evidence.

There are three major frameworks of evidential reasoning in the literature, i.e., the Dempster-Shafer theory of evidence, the fuzzy set theory, and the Bayesian probability theory [1], [2], [3]. The Bayesian model probably is the most popular among the three, which is built on a solid conventional base of probability theory and statistical decision theory. The fuzzy set theory focuses on the issue of representing and managing vague information. Fuzzy logic, in contrast to the conventional possibilistic logic, is used in a variety of approaches which propose a logical treatment of imprecise knowledge referring explicitly to fuzzy set theory. The belief function approach provides a complementary strategy to the Bayesian model. In this approach, the precise specification of a complete probabilistic model is not required, and the conditionalization to represent the impact of a piece of any new evidence is not used any more. In Dempster's formulation, belief functions are interpreted as lower and upper probabilities induced by a family of probability distributions [4]. Shafer then interpreted the Dempster's theory as a model of evidential reasoning [5], [6], [7], [8]. The advantages and weaknesses of these three frameworks have been discussed in [9], [10], [11], [12], and [13].

The application of Shafer's belief function to manage uncertainty of information in a rule-based system has attracted much attention in artificial intelligence research. The Shafer's belief function model uses numerical value in the interval [0, 1] to represent the degree of incompleteness of information. The nonrobustness of this model has been discussed in [14], [15]. Besides this drawback, the basic probability assignment (BPA) of a belief function is in the form of discrete type function which can not always provide a precise description of a piece of evidence for all situations. In many cases, it is not appropriate to assign a discrete probability over [0,1] by thresholding the interval into several regions, since the thresholds themselves can not describe the inexact nature of a piece of evidence. The possible quantization problem caused by thresholding a continuous region for the weight of evidence has been discussed in [9]. The continuous form of belief function, which is a more general representation, is more appropriate for the expression of the vagueness of a piece of evidence in many situations. One example which can not be nicely handled by Shafer's model, or fuzzy sets theory will be given in Section 2.1. Previous belief function approaches also include [16] and [17], which have focused on handling the belief combination problem by using Dempster-Shafer's rule. They didn't provide a formal proof of their method. Other previous related work includes Shafer and Logan's algorithm for hierarchically structured hypotheses [6], and an improved algorithm from the previous one by Shafer and Shenoy [8]. With regard to the dependency relationship between many pieces of evidence, Hau proposed a coefficient between the maximally dependent and independent cases to indicate the degree of dependency [14]. However, the bilateral mutual dependency relationship has not been discussed. Someren presented a learning scheme by using the dependencies among attributes of objects [18]. In a rule-based intelligent system, evidential reasoning often leads to inconsistent results due to the mutual dependency among many pieces of evidence and the structural dependency caused by the improper arrangement of an inference network. One example to illustrate the inconsistency will be given in Example 2 of

Section 4. Such dependency relationship in an inference network has never been seriously addressed in the literature.

The rest of this paper is organized as follows. In Section 2, we present the theory of the proposed belief combination model, and a generalized procedure of belief combination based on the theory. Section 3 gives several examples to demonstrate the advantages of the proposed model. A conclusion is given in Section 4.

2. THEORY OF POTENTIAL MODEL

2.1 Representation of Evidence

The first step in the simulation of human reasoning with uncertainty is to find a proper way to represent the uncertainty and then build up the inference procedure. In the belief function introduced by Shafer [2] and Hau [14], two parameters, i.e., a lower bound and an upper bound, are employed to indicate the *credibility* and the *plausibility*. For the sake of clarity, the *belief function* is borrowed to represent the belief density function associated with an evidence in the following text, and the belief function proposed by Shafer, [2], [6], [8] is named as *Shafer's belief function*. But, as discussed earlier, the probability assignment strategy for Shafer's belief function has its inherent drawback. For example, if a piece of evidence is to emphasize that the closer it is to the truth, the stronger it is, then that evidence can be conveniently modeled by a linear continuous function, which is a density function,

$$Bel(\theta) = k \cdot \theta, \tag{1}$$

where θ is in the interval [0,1] indicating the authenticity of the evidence, and k is a constant. We can hardly find any significant thresholds to quantize the associated belief function into a discrete form which can be handled by either Dempster-Shafer theory [2] or Hau's modified Demspter's rule [14]. Therefore, a more general representation of evidence is needed to represent such kind of uncertainty. In the following, we present our representation of a piece of evidence.

Definition 1: A piece of evidence in a rule-based system is represented by a subset A of the frame of discernment Θ, and a belief function associated with A is represented by a belief density function $p_A(\theta)$, where θ is a variable indicating the degree of truth for the evidence. $\overline{A}$ denotes the complement of A. 1 is used to denote the truth of the evidence and 0 is used to denote the falsity of the evidence. θ is a numerical value in the interval [0,1]. The total amount of belief in the interval [0,1] is

$$\int_0^1 p_A(\theta)d\theta = 1 \tag{2}$$

In order to avoid any confusion, this representation is called **belief density function** which is a function to describe the distribution of a fixed amount of belief, say 1, in an interval [0,1]. This type of belief density function can be transformed into a Shafer's belief function by assigning two bounds to the interval. For instance, if the above linear continuous belief function in Eq. (1) is going to be transformed into the conventional belief function by choosing two thresholds in the belief region [0,1] and compute the respective area of each subregion so that the numerical values of the credibility and the plausibility of this belief function are obtained. If two thresholds, say 1/3 and 2/3, are chosen and k is 2 derived from Eq. (2), the following results are obtained,

$$Cr = \int_{2/3}^{1} 2\theta d\theta = \frac{5}{9}$$

$$Pl = \int_{1/3}^{1} 2\theta d\theta = \frac{8}{9}$$

On the other hand, given a Shafer's belief function by BPA method, e.g., a Shafer's belief function with credibility 5/9 and plausibility 8/9, it can not precisely express the characteristics of the linear continuous belief density function shown by Eq. (1).

The belief density function, $p_A(\theta)$, can also be imagined as a "mass" density function, in which each $p_A(\theta)$ is treated as a point belief located at θ. Each point belief on the evidence will generate a *potential* at a distant location on the hypothesis. The resulted value of the belief function at a certain location on the hypothesis will be the cumulative integral of all the potential generated by all point beliefs on the evidence.

2.2 Spatial interpretation of one evidence's impact

Before we proceed the discussion of belief combination of multiple evidence, the effect of a single evidence onto the hypothesis should be analyzed. Assume a belief function of evidence A is represented by a belief density function, $q_A(\theta)$, on an interval, e.g. $[\overline{A}, A]$, on a plane. A hypothesis C supported by the evidence A is represented by a belief density function, $q_C(\theta)$, on an interval $[\overline{C}, C]$, and is located parallelly to the line segment $[\overline{A}, A]$ at a distance. Line segments $[\overline{A}, A]$ and $[\overline{C}, C]$ has the same length, called **belief length**, usually a unit length. The spatial relationship between the evidence A and the hypothesis C is shown in Fig. 1. The effect of any single point belief of evidence A onto a point on the hypothesis C is determined by two parameters, i.e., the distance between these two points, and the belief strength of the point belief on the evidence A. We can assume that the effect from a point of the evidence A on a point of hypothesis C is proportional to its belief magnitude $q_A(\theta)$ of the point of a piece of evidence, and is inversely proportional to the distance r_{AC} between these two points. To the hypothesis C in Fig. 1, the *potential* $V(\theta)$ at every point θ thereon is affected, respectively, by each point belief $q_A(t)$ on the evidence. This relationship can be described as

$$V(\theta) \propto \frac{q_A(t)}{r_{\mathrm{AC}}} = \frac{q_A(t)}{k \cdot r_{\mathrm{AC}}} \qquad (3)$$

where $r_{AC} = \sqrt{(t-\theta)^2 + R_{AC}{}^2}$, R_{AC} is the vertical distance between line segments, $[\overline{A}, A]$ and $[\overline{C}, C]$, and k is a constant. Hence, the overall contribution of the evidence, A, to a specific degree of belief θ of the hypothesis C can be formulated as following,

$$V_C(\theta) = \int_{\overline{A}}^{A} \frac{q_A(t)}{k \cdot \sqrt{(t-\theta)^2 + R_{AC}{}^2}} dt. \qquad (4)$$

The physical meaning expressed in the last equation is that the "mass" of every point of the evidence will project its effect, which is called **potential**, on every individual point of the hypothesis. As a result, the total effect on a point of the hypothesis is the cumulative sum of the potential from every point of the evidence. The influence of the evidence on the hypothesis can be interpreted by the function of spatial distance and belief strength. Hence, it is necessary to define the relationship of the dependency and the spatial distance more clearly.

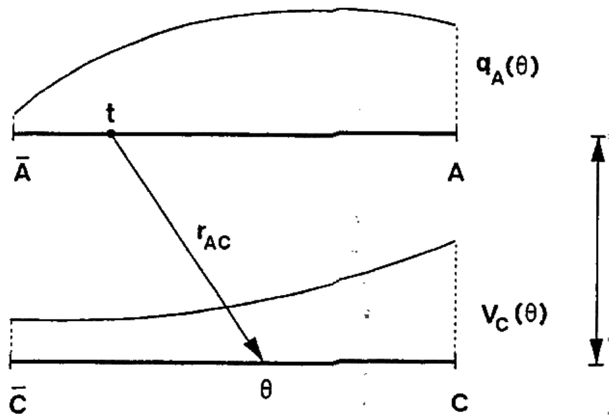


Fig. 1. The relationship between a piece of evidence A and the hypothesis C.

Definition 2: The distance from a piece of evidence to the hypothesis is called its **absolute dependency**, which represents how important this evidence is with respect to the hypothesis.

For example, R_{AC} in Fig. 1 is the absolute dependency of A to C. Two special cases are discussed in the following. We can learn the physical meaning of the dependency in the spatial interpretation from these special cases.

Case 1: $R_{AC} \to \infty$

This special case implies that the evidence "mass" is placed at an infinite distance from the hypothesis, which contributes zero potential to the hypothesis. According to Eqs. (3) and (4),

$$\lim_{r\to\infty} V = \lim_{r\to\infty} \frac{q}{k \cdot r} = 0 \qquad (5)$$

This is an expected result. Its physical meaning is that the evidence has nothing to do with the hypothesis, i.e., they are totally independent.

Case 2 : $R_{AC} \to 0$

In this case, intuitively the physical meaning is that the evidence projects itself onto the hypothesis. This can be shown as follows. Referring to Fig. 1, the ratio of the potentials generated by two mass points, $q_A(t)$ and $q_A(\theta)$, where $t \neq \theta$ onto the position θ on C is

$$\lim_{R_{AC}\to 0}\frac{V_t(\theta)}{V_\theta(\theta)}=\lim_{R_{AC}\to 0}\frac{\dfrac{q_A(\theta)}{\sqrt{(t-\theta)^2+R_{AC}{}^2}}}{\dfrac{q_A(t)}{R_{AC}}}$$

$$=\lim_{R_{AC}\to 0}\frac{q_A(\theta)}{q_A(t)}\cdot\frac{R_{AC}}{\sqrt{(t-\theta)^2+R_{AC}{}^2}}=0$$

This shows that the potential $V_C(\theta)$ is totally determined by the point belief at the position θ of evidence A. Also if there are other evidence with nonzero values of absolute dependencies, their effects on the hypothesis comparatively can be ignored. Hence, the hypothesis is totally dependent upon the evidence A.

Definition 3: The ratio of the absolute dependencies of two beliefs is called their **relative dependency ratio**, which represents the relative importance of the two pieces of evidence with respect to the hypothesis.

For example, the relative dependency ratio of the two pieces of evidence E_1 and E_2 in Fig. 2 can be expressed as

$$\rho_{12}=\frac{R_1}{R_2} \tag{6}$$

Let E_3 be another evidence, then it is easy to show that the following is also true.

Lemma 1: Relative dependency is transitive, i.e.,

$$\rho_{12}=\rho_{13}\cdot\rho_{32} \tag{7}$$

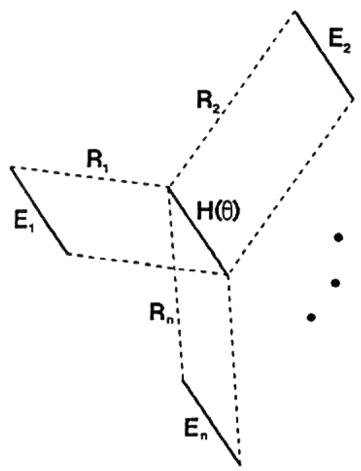


Fig. 2. The relationship between n pieces of evidence, E_i $i=1,\cdots n$, and the hypothesis H.

2.3 Multiple pieces of evidence

We now consider the belief combination which refers to the belief conjunction of several pieces of evidence supporting the same hypothesis.

Definition 4: Belief combination refers to the deduction of the belief associated with $((A\to C)\cap(B\to C))$ from the belief associated with two pieces of evidence A and B, respectively, where C is the hypothesis supported by A and B. That is, given two frames of discernment Θ_A and Θ_B, a compatibility relation between Θ_A and Θ_B is the Cartesian product of them, which is represented as

$$\Theta_A\times\Theta_B\to\Theta_C \tag{8}$$

Consider an example shown in Fig. 3, which has two pieces of evidence A and B. Suppose the relative dependency ratio ρ_{AB} is given. We can set the value of the absolute dependency R_{AC} based on the desired effect of A onto C. Then the total potential distribution of hypothesis C is given by

$$V_C(\theta)=\int_{\overline{A}}^{A}\frac{q_A(t)}{k\cdot\sqrt{(t-\theta)^2+R_{AC}{}^2}}dt+\int_{\overline{B}}^{B}\frac{q_B(t)}{k\cdot\sqrt{(t-\theta)^2+R_{BC}{}^2}}dt \tag{9}$$

where $R_{BC}=\rho_{BA}\cdot R_{AC}$.. Since $V_C(\theta)$only shows a relative degree of the cumulative potential strength of each point on the hypothesis C, it has to be normalized to become a belief density function. The resulted belief density function can then be used in the next stage of belief combination process in an inference network.

Another factor to be taken into consideration is the length of a belief on which a belief density function is distributed. If the length of the line segment on which the belief density function is distributed compared to an absolute dependency is pretty long, then the influence of one end of the evidence will be pretty small to the other end of the hypothesis, e.g., the A to $\overline{C}$ in Fig. 1. Hence the ratio of the length of line segments to a selected absolute dependency will be a factor to the final result of the belief combination.

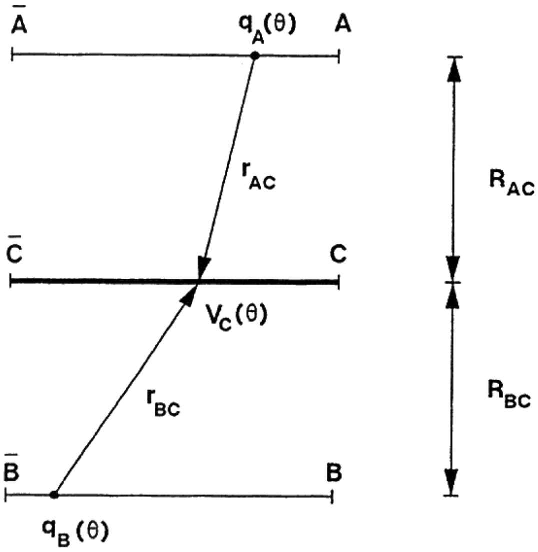


Fig. 3. The relationship between two pieces of evidence A, B and hypothesis C.

Referring to Fig. 3, if a ratio of the length of the belief A, i.e., from A to $\overline{A}$, to its absolute dependency R_{AC} is comparatively large, then the potential generated from one end of the evidence A on the other end of the hypothesis C will be small. Apparently, the final potential distribution on C will depend on this length.

Definition 5: The ratio of the length of the interval, where a belief function is distributed on, with respect to the absolute dependency of the belief density function is called **belief length ratio** (BLR).

For instance, if the BLR = 20, then the length of line segment is 20, and the selected absolute dependency is 1. After the belief combination calculation is done, the belief length is normalized to a unit length. However, this brings up an argument about what a good belief length ratio is in order that a good evidence combination result can be obtained. We have performed some simulations in order to find a reasonably good range of this ratio, which will be presented in Section 3. The following result can be easily proved.

Lemma 2: If the belief length ratio approaches infinite, the potential value of each point on the hypothesis will converge to a constant.

Proof: The potential value of the hypothesis at θ is given by

$$V_C(\theta)=\lim_{L\to\infty}\int_{-L}^{L}\frac{q_A(t)}{k\cdot\sqrt{(t-\theta)^2+R_{AC}{}^2}}dt$$

Let

$$W(t)=\frac{q_A(t)}{k\cdot\sqrt{(t-\theta)^2+R_{AC}{}^2}}\leq\frac{1}{k\cdot\sqrt{(t-\theta)^2+R_{AC}{}^2}}$$

When $t\to\infty, W(t)<\frac{1}{kt}$ as long as $R_{AC}\neq 0$. By elementary calculus, the $V_C(\theta)$ will converge.

In short, this model has some advantages over the previous models:

(1). It can be extended to the combination of many pieces of evidence. In Fig. 2, a model of the combination of n belief density functions is shown. In this case, the resulted value of the belief density function of the hypothesis $V_H(\theta)$ is given as

$$V_H(\theta)=\sum_{i=1}^{n}V_{H_i(\theta)}, \tag{10}$$

where $V_{H_i}(\theta)$ is the potential generated by evidence $E_i, i=1,\cdots,n$.

(2). It provides two parameters for the evidence combination, which allow the mutual dependencies among the evidences and the hypothesis to be easily coped with.

(3). It can handle both the conventional discrete probability assignment and the continuous probability assignment of a belief function.

(4). The physical meaning of dependency, belief strength, and influence of a piece of evidence to a hypothesis can be fully demonstrated in a spatial interpretation.

2.4 A Generalized Procedure for Belief Combination

Based on the above potential model, we propose in the following a procedure for computing the combination of n pieces of evidence. This method can avoid the dependency conflict problem in an inference network. Assume there are n pieces of evidence to be combined, which are expressed as following,

$q_i(\theta)|_{inf}^{sup}, \quad i = 1, \cdots, n$

where $q_i(\theta)$ represents a belief density function distributed on the interval [*inf*, *sup*], and *inf* represents the complement of *sup*. The relative dependency ratio between the *i*th and *j*th beliefs is denoted by ρ_{ij}.

Procedure of combination of multiple evidence

(1) Choose one of the n beliefs as a basic belief, say j, which is supposed to be the strongest evidence supporting the desired hypothesis, i.e., the evidence with the shortest distance to the hypothesis; Set $R_{j,hypo}$ to a desired value.

(2) Compute the potential on the hypothesis by

$$V_{hypo}(\theta) = \sum_{i=1}^{n} \int_{inf}^{sup} \frac{q_i(\theta)}{k \cdot \sqrt{(t-\theta)^2 + (\rho_{ij} R_{j,hypo})^2}} dt \tag{11}$$

where $R_{j,hypo}$ is the absolute dependency of $q_j(\theta)$ to the hypothesis, and (*sup* - *inf*) = belief length .

(3) Normalize $V_{hypo}(\theta)$ to a belief density function form.

Note that a rule of thumb of selecting the basic belief is to choose the evidence having the strongest absolute dependency with respect to the hypothesis.

3. Examples of Simulation

In this section, the feasibility and advantages of the proposed potential model are demonstrated by several examples.

Example 1.

This example shows the capability of the proposed model to handle the belief combination of discrete type belief density functions. Suppose the uncertainty of a belief density function is distributed on the [0,1] interval, where 0 denotes the complete false, and 1 the complete true. Assume two pieces of evidence, A and B, have the relative dependency $\rho_{BA} = 0.6$, which implies the evidence A is the stronger of the two pieces of evidence with respect to the hypothesis C. Let

$$p_A(\theta) = 0.3\delta(\theta - 0.0) + 0.4\delta(\theta - 0.5) + 0.3\delta(\theta - 1.0)$$

$$p_B(\theta) = 0.2\delta(\theta - 0.0) + 0.4\delta(\theta - 0.5) + 0.4\delta(\theta - 1.0),$$

which are illustrated in Fig. 4. Following the procedure given in the previous section, the absolute dependency R_{AC} is chosen as a unit, i.e. $R_{AC} = 1$. Hence, the absolute dependency R_{BC} is determined by Eq.(6),

$$R_{BC} = \frac{1.0}{0.6} = 1.666$$

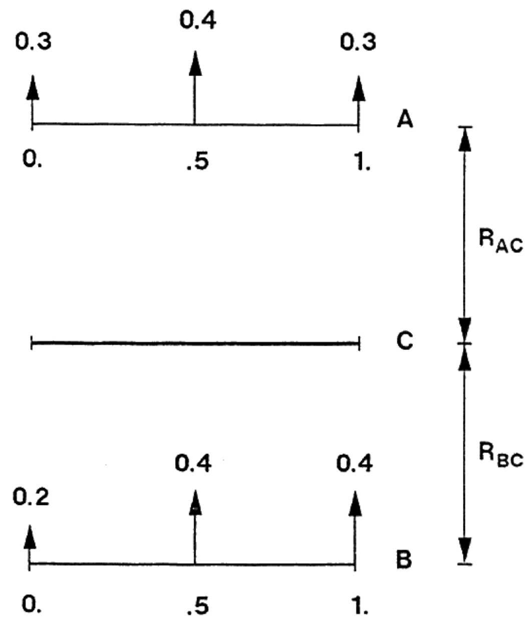


Fig. 4. An example to demonstrate the proposed belief combination model.

The belief density function of hypothesis C is then computed by applying the procedure of Section 2.2. The resulted credibility and plausibility of belief function $p_C(\theta)$ of the hypothesis C are tabulated in Table 1, where the Θ_C represents the unknown portion of belief of the hypothesis C, Cr_c the credibility, and Pl_C the plausibility.

Here, we have employed two conventional notations which are the *Cr*, *credibility*, referring to the lower bound of the belief function defined in [2] and [14], and the *Pl*, *plausibility*, referring to the upper bound, for the sake of comparison. From Table 1, we can see that when the BLR approaches infinite, the belief density function of hypothesis C will converge to a stable value, which is predicted by Lemma 2. We can also see that if the BLR is small, which indicates that the two ends of belief function are close to each other, then the mutual interaction of point beliefs is strong, and vice versa. From the simulation result, a reasonable choice of a belief length ratio seems to be about 20 to 100.

BLR	Cr_C	Θ_C	$1 - Pl_C$
1	0.328649	0.346598	0.324753
2	0.325444	0.360448	0.314108
5	0.325997	0.377129	0.296874
10	0.329302	0.385782	0.284916
20	0.332534	0.391732	0.275735
50	0.335258	0.396312	0.268430
100	0.336332	0.398081	0.265587
200	0.336903	0.399021	0.264076
500	0.337258	0.399603	0.263138
1000	0.337379	0.399801	0.262821

Table 1: The results of proposed model applied to Fig. 4.

Example 2.

In [14], one very crucial problem was pointed out, which is that in a sequential programming rule-based system, the structural dependency problem can hardly be avoided. Referring to Fig. 5, suppose there are three pieces of uncertain evidence to be combined. In a sequential programming style, two of them have to be combined first, then the result of this combination will be combined with the third one. Two possible structures of combination are shown in Fig. 6. If all of these three pieces of evidence are totally independent with each other, there will be no conflict in the final result for the two structures in Fig. 6. But if these three pieces of evidence are partially dependent upon one another, then conflict will happen. The final results obtained from the two different structures will be inconsistent with each other.

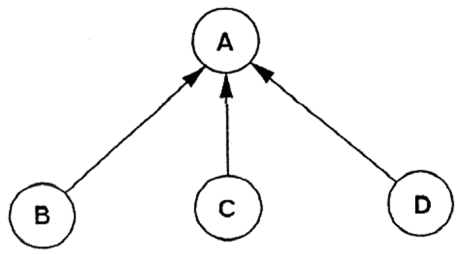
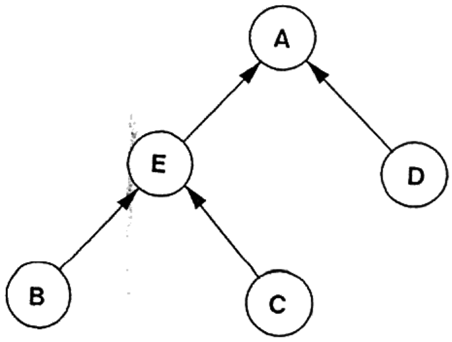


Fig. 5. Combination of three pieces of evidence B, C, and D to support the hypothesis A.

Case 1:



Case 2:

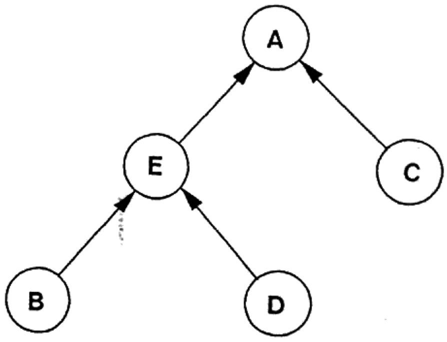


Fig. 6. Two different structures of combination of the three pieces of evidence B, C, and D.

Consider the case in Fig. 5 and the two different structures of combination shown in Fig. 6. Assume

$$Cr(B)=0.98,\quad Pl(B)=0.99,\quad \rho_{CD}=\rho_{DC}=0.5,$$
$$Cr(C)=0.01,\quad Pl(C)=0.02,\quad \rho_{CB}=0.1,$$
$$Cr(D)=0.01,\quad Pl(D)=0.99,\quad \rho_{DB}=0.9$$

Based on Hau's approach [14], the results listed in Table 2 will be obtained. There are some obvious inconsistencies in the results of Table 2. From the assumption, evidence B is the strongest one to support the hypothesis A, the other two pieces of evidence C and D are less important than B. According to Table 2, the resulted credibility of case 1 is more than twice of that of case 2. On the contrary, the plausibility, $(Cr_A + \Theta_A)$ of case 1 is only half of that of case 2. These results indicate that Hau's method is easily subject to the combination order of the evidence, which is not consistent with the intuition of human reasoning. According to intuition, if these three pieces of evidence are given, a belief function associated with A should be dominated by B, since the relative dependency ratios of C and D indicate their less influence on A, and because D has a stronger dependency to B than C, B should have the dominant impact on A. However, the result in Table 2 does not show that B has the dominant influence on A. Therefore, we conclude that the results given by Hau's method are not consistent with the human reasoning. The reason why the conflict appears in this example is that the mutual dependency relationship will *propagate through the inference network*. Our model, on the other hand, can resolve this problem and provide a much more reasonable result than the conventional approaches. As a comparison, we use our model to compute the belief function of hypothesis A and the results are listed in Table 3.

	Cr_A	Θ_A	$1-Pl_A$
Case 1	0.006903	0.012857	0.980239
Case 2	0.003064	0.033357	0.963579

Table 2: The results of Hau's approach applied to either case of Fig. 6.

BLR	Cr_A	Θ_A	$1-Pl_A$
1	0.352041	0.348043	0.299916
2	0.375143	0.363695	0.261162
5	0.414667	0.391576	0.193757
10	0.441216	0.414554	0.144230
20	0.460307	0.431718	0.107975
50	0.477805	0.441651	0.080544
100	0.485792	0.444197	0.070011
200	0.490254	0.445354	0.064391
500	0.493070	0.446039	0.060891
1000	0.494030	0.446269	0.059702

Table 3: The results of proposed model applied to case of Fig. 5.

Based on Table 2 and Table 3, a comparison is given as follows. Evidence B is the strongest one to support the hypothesis A, the other two pieces of evidence C and D are less important than B. Hence, a belief function associated with A should be dominated by B, since the relative dependency ratios of C and D indicating their less influence to A. Also the result should be consistent, despite the arrangement of the inference network. Referring to Table 3, no matter what the belief length is, the credibility Cr_A and plausibility $(Cr_A+\Theta_A)$ of hypothesis A are strongly influenced by B, and only slightly perturbed by C and D. This is close to what we expect from the intuition.

Example. 3

This example is going to illustrate the *capability of the proposed model to handle the* uncertainty aggregation of continuous belief functions. Suppose there are two pieces of evidence A and B which both support a hypothesis C. We are given the following information,

$$p_A(\theta)=2\theta, \quad \theta\in[0,1]$$

$$p_B(\theta)=\begin{cases}1.0, & 0\le\theta\le\frac{1}{3}\\ 0.25, & \frac{1}{3}\le\theta\le\frac{2}{3}\\ 1.75, & \frac{2}{3}\le\theta\le 1\end{cases}$$

$$\rho_{BA}=0.5$$

where $p_A(\theta)$ and $p_B(\theta)$ are the belief density functions associated with two pieces of evidence A and B, respectively. The graphs of the above belief functions are shown in Fig. 7. From the given information and Fig. 7, we are expecting a reasonable result of the belief combination of these two pieces of evidence which should be dominated by the stronger evidence A and perturbed by the weaker evidence B. By applying the procedure given in Section 2.4, we obtain the results which are a family of belief functions of the hypothesis C for different belief length. If two thresholds, which are 0.333 and 0.667, in the interval [0,1] are selected to quantize the belief function into the conventional form with a credibility and a plausibility, then we have the results listed in Table 4.

From Fig. 7 and Table 4, it is obvious that no matter what the belief length ratio is chosen, the belief density function of the hypothesis C is dominated by the stronger evidence A, and perturbed by the weaker evidence B. When the belief length is small, the point beliefs close to one end of the belief density function of the evidence have great influence on the points of the other end of the hypothesis; and vice versa. Therefore, when the belief length ratio is getting larger, the impact by the weaker evidence will gradually appear. The results shown in Fig. 7 meet what we anticipate.

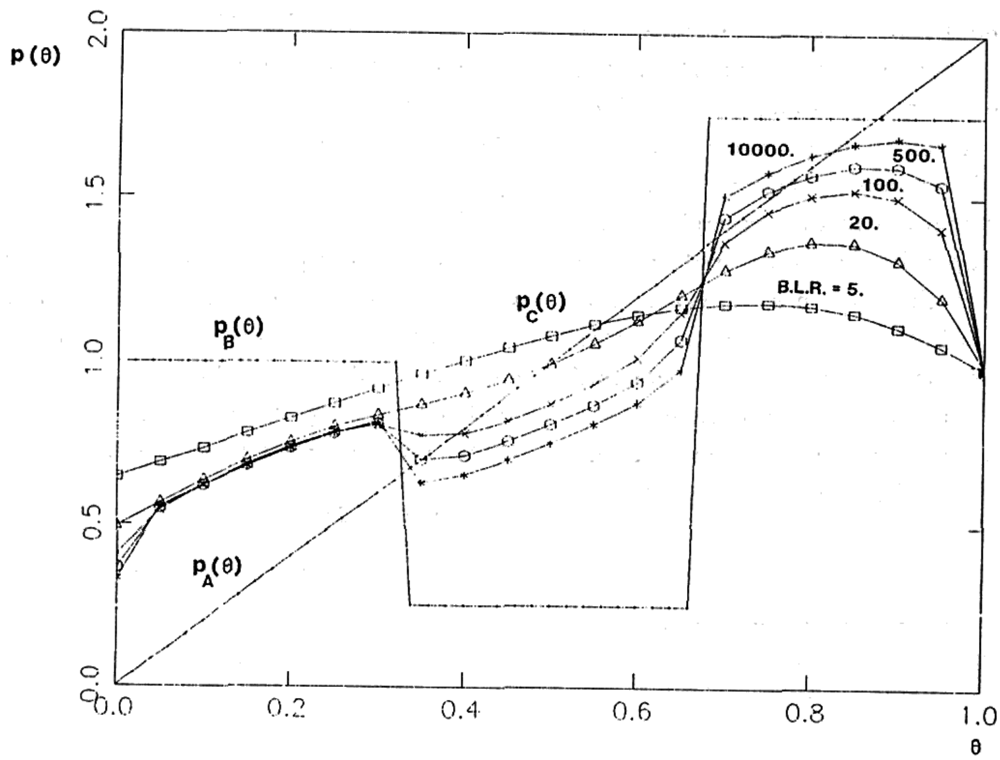


Fig. 7. Simulation results of Example 3.

BLR	*Cr*	*Pl*
5	0.395806	0.765112
20	0.447528	0.793486
100	0.491115	0.800420
500	0.514544	0.801611
10000	0.535527	0.802300

Table 4: The results of proposed model applied to Example 3.

4. Conclusion

A new approach to handle uncertainty aggregation of belief combination in rule-based systems is presented in this paper. Reasoning with uncertainty in a rule-based system is considered as the aggregation of uncertain information about the belief from different sources. Belief combination is one type of aggregation of uncertain information, and is a critical operation of information fusion. If Dempster's rule [2], [6] or Hau's approach [14] is adopted, the conflict resulting from the structural dependency of a lattice-structured inference network can not be resolved. Our proposed approach, on the other hand, provides a remedification to this conflict resolution problem.

Our method offers several advantages over previous methods. First, the conflict due to the mutual dependency relationship among different pieces of evidence in an inference network is solved. Second, not only the discrete belief functions, but also the arbitrary continuous belief density functions can be handled, which has not been explored up to date. The merit of a continuous belief density function is that it can better represent the vagueness of a human concept than a conventional discrete one. Third, given different belief length ratios, different potentials of the goal will be obtained, which provides a variety of options. The proposed model also provides an interpretation for belief combination by a spatial view of evidence and hypothesis. Fourth, the complexity of computation in Dempster's belief function approach or Hau's approach in a inference network is significantly reduced because the proposed model avoids the inherent structural dependency problem. The simulation results of the proposed model also turn out to be more appealing.

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