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Fuzzy data processing using polynomial bidirectional hetero-associative network

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Abstract

This investigation presents a novel method of fuzzy data processing using polynomial bidirectional hetero-associative network (PBHAN). This has a higher capacity for pattern pair storage than that of the conventional bidirectional associative memories (BAMs) and fuzzy memories. In addition, a new energy function is defined. The PBHAN takes advantage of fuzzy characteristics in evolution equations such that the signal-noise-ratio (SNR) is significantly increased. The energy of the PBHAN defined by the proposed energy function decreases as the recall process proceeds, thereby ensuring the stability of the system. In this work, we prove the stability of fuzzy data processing using PBHAN. The increase of SNR consequently enhances the capacity of the PBHAN. The capacity of the fuzzy data processing using PBHAN in the worst case is also estimated. © 2000 Elsevier Science Inc. All rights reserved.

Keywords: Polynomial bidirectional hetero-associative network (PBHAN); SNR; Stability

1. Introduction

Associative memories have received extensive interest in neural networks [1–6,9,10]. In related works, Kosko [7,8,11–13] presented a fuzzy associative memory (FAM) system structure. Kosko's FAM is the very first example to use neural networks to articulate fuzzy rules for fuzzy systems. The FAM model

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has been successfully applied to problems like target tracking [16], backing up a truck-and-trailer [18], and voice-cell control in ATM networks [17] where distinctive features like robustness, modularity, and adaptability have been demonstrated. However, no energy function introduced in their works could ensure that every stored pattern pair resides at a local minimum on energy surfaces. Moreover, no capacity analysis was performed as well. Despite its simplicity and modularity, this model suffers from extremely low-memory capacity, i.e., one rule per FAM matrix. Besides, it is limited to small rule-based applications.

There has been a renewal of interest in FAMs in recent years. For instance, Yamaguchi et al. [20] presented a method to represent fuzzy IF-THEN rules using associative memories and carry out fuzzy inference using association; a conceptual fuzzy set (CFS) comprised of distributed fuzzy knowledge processing have been proposed by Takagi et al. [21]. However, it is difficult to apply FAM to complex knowledge processing, because these associative memories have very poor storage capacity. Chung and Lee [15,19] proposed a multiple-rule storage property of a FAM matrix. They showed that more than one rule can be encoded by Kosko's FAM. However, they did not derive the maximum capacity of a FAM. The actual capacity will depend on the dimension of the matrix and the rule characteristics, e.g., how many rules are overlapped. The capacity of this model suffers from the limitations since the capacity depends on whether the membership function is semi-overlapped or not.

The aims of this paper include:

1. an attempt to overcome the poor capacity in the works of other investigators;
2. a means of developing the mathematical theory associated with PBHAN.

A perfect recall theorem is established in this paper and the implementation of the PBHAN model is more efficient accordingly. Firstly, we analyze the framework of the high-capacity PBHAN in which the component of a fuzzy vector is termed a fuzzy bit (**fit**). Moreover, we propose our energy function and a two-phase approach to verify the stability. We adopt the signal-noise-ratio (SNR) approach to derive the equations of the sufficient condition of the PBHAN and, thus, attain the Z value, which is the power of polynomial, and capacity of the PBHAN. The smallest Z value, which can still recall all of the stored pattern pairs such that the dimension of the patterns can be as large as possible, is also attained. Any Z value, which satisfies the condition of the derived absolute lower bound, can recall all of the different stored patterns. Finally, the capacity of PBHAN in the worst case is also derived.

2. Framework of high-capacity PBHAN

2.1. Evolution equations

Assume that we are given M pattern pairs, which are

$$\{(X_1, Y_1), (X_2, Y_2), \ldots, (X_M, Y_M)\}, \tag{1}$$

where

$$X_i = (x_{i1}, x_{i2}, \ldots, x_{in}), \qquad Y_i = (y_{i1}, y_{i2}, \ldots, y_{ip}).$$

Let $1 \leqslant i \leqslant M$, $x_{ij} \in [0, 1]$, $1 \leqslant j \leqslant n$, $y_{ij} \in [0, 1]$, $1 \leqslant j \leqslant p$, n and p are the component dimensions of X and Y, and n is assumed to be smaller than or equal to p without any loss of generality. x_{ik}, $y_{ik} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$, fuzzy space = [1, 0], λ is a fuzzy quantum, and σ is a fuzzy quantum gap. Instead of using Kosko's [10] approach, we use the following evolution equations in the recall process of the PBHAN:

$$y_k = \frac{\sum_{i=1}^{M} y_{ik} \cdot \left(\frac{u - \|X_i - X\|^2}{u}\right)^{M^z}}{\sum_{i=1}^{M} \left(\frac{u - \|X_i - X\|^2}{u}\right)^{M^z}}, \tag{2}$$

$$x_k = \frac{\sum_{i=1}^{M} x_{ik} \cdot \left(\frac{u - \|Y_i - Y\|^2}{u}\right)^{M^z}}{\sum_{i=1}^{M} \left(\frac{u - \|Y_i - Y\|^2}{u}\right)^{M^z}}, \tag{3}$$

where M denotes the number of patterns in the PBHAN; X_i, Y_i, $i = 1, \ldots, M$, represent the stored patterns; X or Y is the initial vector presented to the network; x_k and x_{ik} denote the kth digits of X and X_i, respectively; y_k and y_{ik} represent the kth digits of Y and Y_i, respectively; Z is a positive integer; u denotes a function defined as

$$u = \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\|X_i - X_j\|^2 + \|Y_i - Y_j\|^2 \right). \tag{4}$$

Notably, u is bounded according to Eq. (4).

2.2. Energy function and stability

The fact that every stored pattern pair should produce a local minimum on the energy surface to be recalled correctly accounts for why the energy function is intuitively defined as

$$E(X, Y) = \sum_{i=1}^{M} \|X - X_i\|^2 \cdot \|Y - Y_i\|^2. \tag{5}$$

Fuzzy data model using PBHAN can be deemed as a variety of bidirectional associative memory (BAM) [14]. Therefore, its stability can be elucidated by closely examining its two phases of evolution, i.e., $X \rightarrow Y$ and $Y \rightarrow X$.

Theorem 1. *The PBHAN modeled by* [Eqs. (2) *and* (3)] *is a stable system.*

Proof. We discuss the stability by observing the behavior of energy function of two directions, $X \to Y$ and $Y \to X$, respectively.

Phase 1: $X \to Y$. We use the energy function as Eq. (5). Thus, the $\nabla_{x_k} E(X, Y)$ can be computed as

$$\begin{aligned}\nabla_{x_k} E(X,Y) &= 2\sum_{i=1}^{M}(x_k - x_{ik})\|Y - Y_i\|^2 \\ &= 2\left(\sum_{i=1}^{M}\|Y - Y_i\|^2\right) \\ &\quad\cdot\left[x_k - \frac{\sum_{i=1}^{M} x_{ik}\left((u - \|Y - Y_i\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u - \|Y - Y_i\|^2)/u\right)^{M^z}}\right].\end{aligned} \tag{6}$$

The difference of E due to a **fit**'s change can, therefore, be derived as

$$\begin{aligned}\Delta_{x_k} E(X,Y) &= \nabla_{x_k} E(X,Y)\cdot \Delta_{x_k} \\ &= -2\left(\sum_{i=1}^{M}\|Y - Y_i\|^2\right) \\ &\quad\cdot\left[x_k - \frac{\sum_{i=1}^{M} x_{ik}\left((u - \|Y - Y_i\|^2)/u\right)^{M^Z}}{\sum_{i=1}^{M}\left((u - \|Y - Y_i\|^2)/u\right)^{M^Z}}\right]\cdot(x'_k - x_k) \\ &= -2\left(\sum_{i=1}^{M}\|Y - Y_i\|^2\right) \\ &\quad\cdot\left[\frac{\sum_{i=1}^{M} x_{ik}\left((u - \|Y - Y_i\|^2)/u\right)^{M^Z}}{\sum_{i=1}^{M}\left((u - \|Y - Y_i\|^2)/u\right)^{M^Z}} - x_k\right]\cdot(x'_k - x_k).\end{aligned} \tag{7}$$

According to Eq. (3), we have the following inequalities when x'_k is the next state of x_k.

Case 1: If

$$x_k - \frac{1}{2\lambda} \leqslant \frac{\sum_{i=1}^{M} x_{ik}\left((u - \|Y - Y_i\|^2)/u\right)^{M^Z}}{\sum_{i=1}^{M}\left((u - \|Y - Y_i\|^2)/u\right)^{M^Z}} \leqslant x_k + \frac{1}{2\lambda},$$

then $x'_k = x_k$ according to Eq. (3). Thus, $\Delta_{x_k} E(X, Y) = 0$.

Case 2: If

$$\frac{\sum_{i=1}^{M} x_{ik}\left((u-\|Y-Y_i\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u-\|Y-Y_i\|^2)/u\right)^{M^z}} \geqslant x_k + \frac{1}{2\lambda},$$

then $x'_k > x_k$. Thus, according to Eq. (7), $\Delta_{x_k} E(X,Y) < 0$.

Case 3: If

$$\frac{\sum_{i=1}^{M} x_{ik}\left((u-\|Y-Y_i\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u-\|Y-Y_i\|^2)/u\right)^{M^z}} < x_k - \frac{1}{2\lambda},$$

then $x'_k < x_k$. Thus, according to Eq. (7), $\Delta_{x_k} E(X,Y) < 0$.

In sum, $X \to Y$ phase causes E to decrease, $\Delta_{x_k} E(X,Y) \leqslant 0$.

Phase 2: $Y \to X$. By the similar derivation as shown in the $X \to Y$ phase, we also can prove that $Y \to X$ is asymptotically stable. The only difference is the definition of the energy function in this phase. We again use the energy function as Eq. (5). Thus, the $\nabla_{y_k} E(X,Y)$ can be computed as

$$\begin{aligned}\nabla_{y_k} E(X,Y) &= 2\sum_{i=1}^{M}(y_k - y_{ik})\|X-X_i\|^2 \\ &= 2\left(\sum_{i=1}^{M}\|X-X_i\|^2\right) \\ &\quad\cdot\left[y_k - \frac{\sum_{i=1}^{M} y_{ik}\left((u-\|X-X_i\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u-\|X-X_i\|^2)/u\right)^{M^z}}\right]. \end{aligned} \tag{8}$$

Since the procedure of the derivation is very much the same as that of the $X \to Y$ phase, there is no need to repeat the lengthy discussion.

Notably, the energy function defined in Eq. (5) is bounded. Meanwhile, the $X \to Y$ phase always drags down $E(X, Y)$, while the $Y \to X$ phase also reduces $E(X, Y)$. Then, the evolution will be terminated when $E(X, Y)$ reaches a minimum, where a pattern pair is stored. □

2.3. Analysis of capacity of PBHAN

The SNR approach is adopted herein to compute the capacity of PBHAN. Let X_i and Y_i be the stored pattern pairs. Assume that X_1 is the input pattern pair and Y_1 is recalled expectantly. Substituting X_1 for X allows us to rewrite Eq. (2) as

$$y_k \cdot \sum_{i=1}^{M} \left(\frac{u - \|X_i - X_1\|^2}{u} \right)^{M^z} = y_{1k} \cdot 1^2 + y_{2k} \cdot \left(\frac{u - \|X_2 - X_1\|^2}{u} \right)^{M^z} + y_{3k} \cdot \left(\frac{u - \|X_3 - X_2\|^2}{u} \right)^{M^z} + \cdots + y_{Mk} \cdot \left(\frac{u - \|X_M - X_1\|^2}{u} \right)^{M^z}. \quad (9)$$

The largest noise that can appear is in the worst case which any X_i, $i \neq 1$, is just one component different from X_1. Meanwhile, the other components of X_i and X_1 remain the same. For instance, $X_1 = (x_{11}, x_{12}, \ldots, x_{1n})$, and $X_i = (x_{11}, x_{12}, \ldots, x_{1n} \pm 1/(2\lambda))$, $i \neq 1$, where $x_{ik} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$, $k = 1, 2, \ldots n$, and $y_{ik} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$, $k = 1, 2, \ldots p$. The first term in the above equation corresponds to the signal, and the other terms are the noise. The power of signal is

$$S = y_{1k} \cdot 1^2. \quad (10)$$

Besides the first term, the remaining terms are actually the sum of $M - 1$ independent identically distributed random variables. Therefore, the noise of these terms is $M - 1$ times of the noise of a single random variable. The following inequalities can be obtained from Eq. (9).

$$y_k \leqslant y_k \cdot \sum_{i=1}^{M} \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^z} \leqslant y_{1k} \cdot 1^2 + \sum_{j=2}^{M} y_{jk} \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} \leqslant y_{1k} \cdot 1^2 + (M-1) y_{2k} \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} = S + N_{\max}. \quad (11)$$

The second term is viewed as the total noise in the worst case. Let $y_{2k} = j/\lambda$, $j \in \{0, 1, 2, 3, \ldots, \lambda\}$. The sufficient condition for the noise must be bounded is

$$y_{1k} - \frac{1}{2\lambda} < y_{1k} \cdot 1^2 + (M-1) y_{2k} \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} < y_{1k} + \frac{1}{2\lambda}. \quad (12)$$

Herein, we take $y_{2k} = j/\lambda$ into the above equation, then it can be simplified as:

$$
\begin{aligned}
-\frac{1}{2\lambda} &< (M-1)\cdot\frac{j}{\lambda}\cdot\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z} < \frac{1}{2\lambda},\\
-\frac{1}{2} &< (M-1)\cdot j\cdot\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z} < \frac{1}{2},\\
&\left|(M-1)\cdot j\cdot\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z}\right| < \frac{1}{2},\\
&\left|(M-1)\cdot\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z}\right| < \frac{1}{2j} \leqslant \frac{1}{2}.
\end{aligned}
\tag{13}
$$

The worst case will occur when $j = 1$. Thus, we deem the above equation as the sufficient condition for the PBHAN to accurately recall any pattern. Since the value inside of the absolute bracket in Eq. (13) is positive, the minimal Z in the worst case for the PBHAN is derived in the following:

$$
\begin{aligned}
(M-1)\cdot\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z} &\leqslant \frac{1}{2},\\
\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z} &\leqslant \frac{1}{2(M-1)},\\
\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z} &\leqslant \frac{1}{2},\\
\ln\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z} &\leqslant \ln\left(\frac{1}{2}\right),\\
M^z\cdot\ln\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)^{M^z} &\leqslant \ln\left(\frac{1}{2}\right).
\end{aligned}
\tag{14}
$$

Since the second product term on the left-hand side of Eq. (14) is smaller than zero, we obtain:

$$
M^z \geqslant \frac{\ln(1/2)}{\ln((u-(1/\lambda^2))/u)},
\tag{15}
$$

$$
\begin{aligned}
\ln M^z &\geqslant \ln\left\{\frac{\ln(1/2)}{\ln((u-(1/\lambda^2))/u}\right\},\\
Z &\geqslant \frac{1}{\ln M}\cdot\ln\left\{\frac{\ln(1/2)}{\ln((u-(1/\lambda^2))/u}\right\},
\end{aligned}
\tag{16}
$$

where

$$u \leqslant \mathscr{C}_2^M \cdot (n+p) \leqslant \mathscr{C}_2^M \cdot (2n) \leqslant M \cdot (M-1) \cdot n. \tag{17}$$

Since we wish to derive the absolute upper bound of u, it would be reasonable to use $n = \max(n,p)$ instead of $n = \min(n,p)$ in the result of Eq. (17). Hence, the Eq. (16) is the lower bound solution of Z, and according to Eq. (15), the capacity can be derived as

$$M \geqslant \left\{ \frac{\ln(1/2)}{\ln((u-(1/\lambda^2))/u)} \right\}^{Z^{-1}}, \tag{18}$$

where

$$u = M \cdot (M-1) \cdot n.$$

3. Simulation analysis

To verify the capacity analysis described in Section 2.3, we utilize computer programs to produce the values among M, Z, and n for λ equal to 2, 5, 10, and 100 for the lower bound of Z. Figs. 1–4 plot the above results, respectively. (The legends represent the values of Z). After the lower bound solution of Z is derived, the capacity of PBHAN can be computed by Eq. (18). Figs. 5–7 are the relationships of capacity, M, vs. n. In Fig. 5, $\lambda = 10$, $Z = 3$; in Fig. 6, $\lambda = 100$, $Z = 4$; in Fig. 7, $\lambda = 300$, $Z = 4$. According to these figures, the PBHAN provides a significantly high-capacity of storage for pattern pairs.

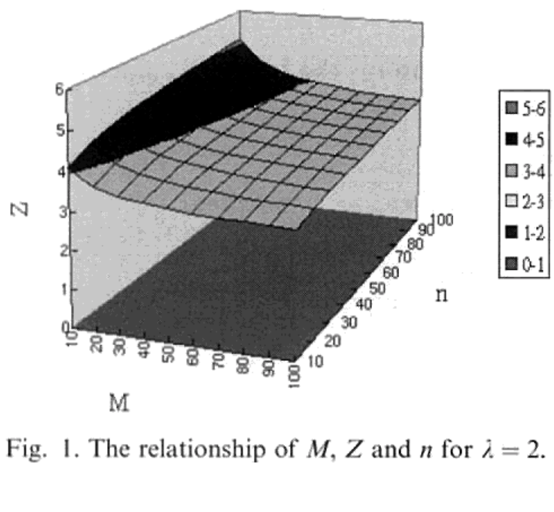

Fig. 1. The relationship of M, Z and n for $\lambda = 2$.

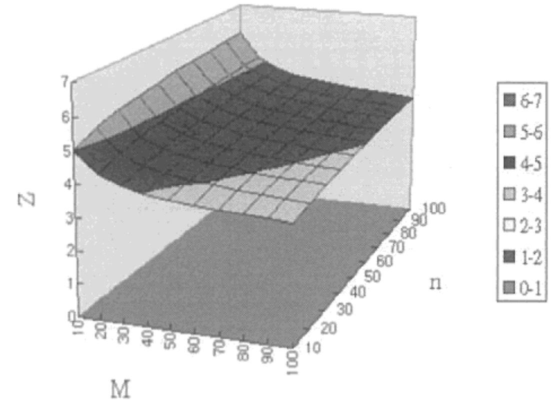

Fig. 2. The relationship of M, Z and n for $\lambda = 5$.

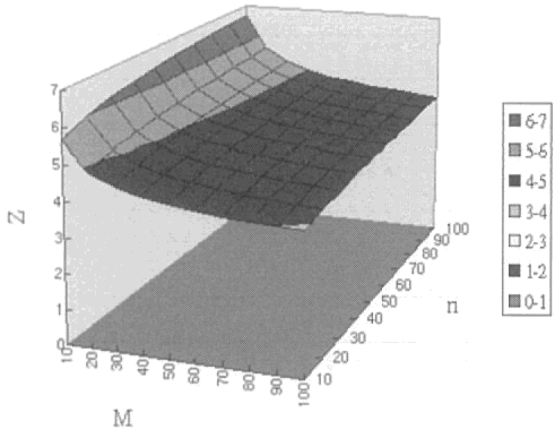

Fig. 3. The relationship of M, Z and n for $\lambda = 10$.

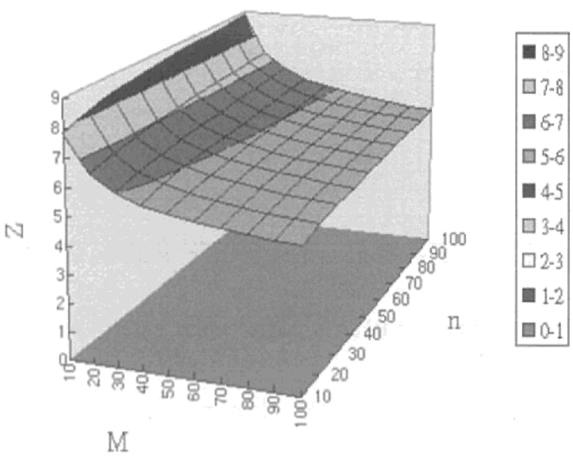

Fig. 4. The relationship of M, Z and n for $\lambda = 100$.

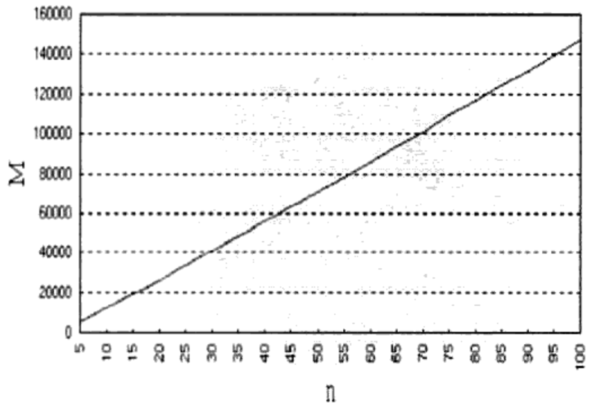

Fig. 5. The capacity of PBHAN in the worst case ($\lambda = 10,\ Z = 3$).

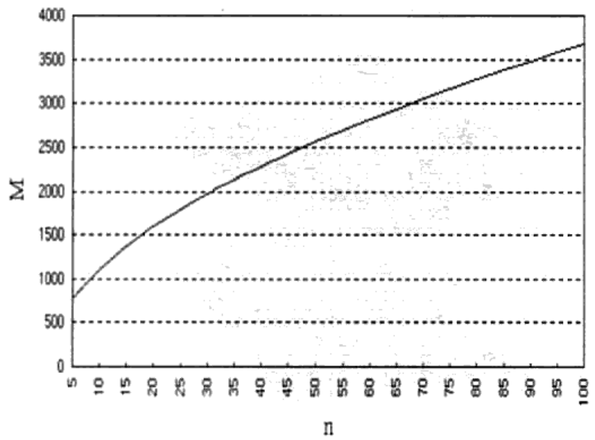

Fig. 6. The capacity of PBHAN in the worst case ($\lambda = 100,\ Z = 4$).

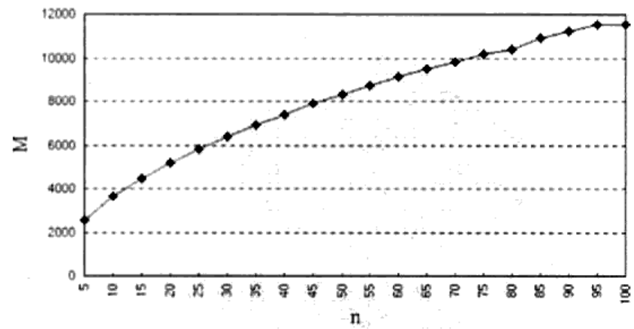

Fig. 7. The capacity of PBHAN in the worst case ($\lambda = 300,\ Z = 4$).

Example 1. We can use the result of this research for pattern recognition problems. The PBHAN is used to store and recall a set of 7×11 fuzzy data composed of 26 different pattern pairs (English letters, upper case and lower

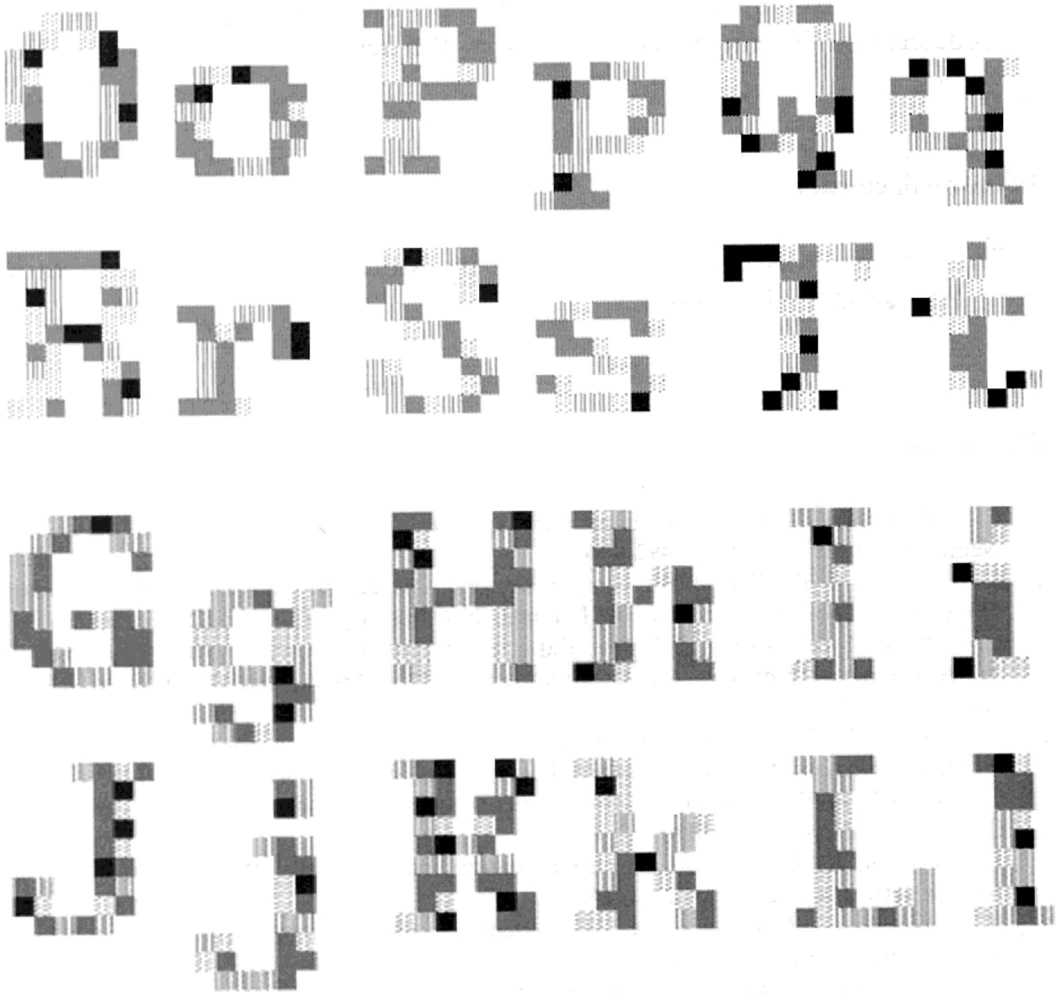

Fig. 8. Pattern recognition examples ($M = 26$, $n = p = 77$).

case). We then apply the evolution equation in Section 2.1. Fig. 8 presents some pattern pairs with $n = p = 77$ to this network. The number of these pattern pairs are much less than our storage capacity, (M), thus, according to our simulation result, only one iteration is required for every capital letter to recall its corresponding lower case letter correctly, and vice versa.

4. Conclusion

According to our results, the PBHAN provides an extremely high-storage capacity for pattern pairs. This method utilizes a polynomial scheme to magnify the capacity. The proposed energy function ensures that every stored pattern pair is located in a local minimum of the energy surface. The capacity of the PBHAN in the worst case is analytically estimated, thereby allowing us to predetermine the size of the PBHAN by the demand of capacity possible.

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